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Residual stresses arising in materials deformed by bending in surface-active media

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The distribution of residual stresses in rectangular beams after loading by clear bending in inactive, surface-plasticizing and surface-hardening media was investigated. It was found that the residual stresses in a more plastic structural component are typically opposite in sign to active load stresses, whereas in a harder component they are coincident with the latter. The influence of residual stresses on the point defect distribution in the samples after deformation is discussed.

1. Introduction

In solving some problems of physico-chemical mechanics it is necessary to estimate residual stresses arising in the material after being loaded in surface-active media. In many practically important cases, the complex stress state occurring in the material under load and the trajectory of its release can be represented as a superimposition of elementary single-axis and planar approximations [1]. In this connection it is obvious that the theoretical analysis should be applied to the residual stresses arising in the material after the deformation process in active media when considering such elementary cases as clear bending.

According to the numerous experimental data [2, 3], the plasticizing of a sample surface, i.e. lowering of the yield stress point of its surface layer, can be considered as a consequence of the adsorption of individual molecules of active media at the solid surface, just the adsorption causes the reducing of the free surface-energy level and facilitating the work of the near-surface dislocation sources. On the contrary, the formation of thinnest adsorptive, oxide and passivating films, which fix the ends of the near-surface dislocations, impede their release from the crystal and inhibit the work of dislocation sources, results in hardening of the sample surface, i.e. a raising of the yield stress point of its surface layer [3, 4]. In each of the two cases mentioned, the thickness of the external modified layer under consideration depends on the media activity and the sample defect structure. The experience shows that the change in plastic characteristics in many construction materials can be observed in the near-surface layers down to several micrometres and even tens of micrometres [2–4].

This paper reports a study of the behaviour of a model isotropic ideally elasto-plastic (not hardened under deformation) material which has equal cross-sectional values of plastic characteristics in the absence of any media. The effect of an active environment was taken into account by introducing a solid layer of restricted thickness with the yield stress point, σ_{ss}, differing from that of the inner bulk, σ_s. The boundary between the near-surface layer and the inner bulk zone was considered to be infinitely thin, and the effect of the environment on the elastic characteristics was neglected. The clear elasto-plastic bending of a symmetric beam in the Cartesian coordinates connected with the neutral section was analysed (Fig. 1).

2. Homogeneous beam (tests in an inert environment: $\sigma_{ss}/\sigma_s = 1$)

The general solution of the problem of the distribution of stresses in a homogeneous beam subjected to a clear elasto-plastic bending is known (see, for example, [5]). The residual stress distribution after bending around the $0Y$ axis (Fig. 2a) appears as

$$\sigma^r_{ij} = 0 \qquad \text{when } i, j \neq z \tag{1a}$$

$$\sigma^r_{zz}(x) = \begin{cases} \pm\,\sigma_s - \dfrac{M_0}{J}x & \text{when } C_0 \leqslant |x| \leqslant b \\ \left(\dfrac{\sigma_s}{C_0} - \dfrac{M_0}{J}\right)x & \text{when } |x| \leqslant C_0 \end{cases} \tag{1b}$$

where $J = 2ab^3/3$ is the inertia moment of the beam cross-section; the upper sign refers to the beam side stretched by the external forces (in Fig. 2a, when $x > 0$), the lower sign refers to the compressed side (when $x < 0$).

The residual stresses in the near-surface layer of the beam are opposite in sign to the external stresses caused by the bending moment, M_0; their signs are the same in the inner layer. The residual stresses change their sign in those sections of the beam which are parallel to the neutral one ($x = 0$) and they are situated at the distances of $x_0 = \pm\,\sigma_s J/M_0$ from it. The residual stresses attain their maximum values at the beam surface (when $x = \pm\,b$) as well as in the sections parallel to the neutral one and situated on each side of it at the distances $x = \pm\,C_0$. The value C_0 characterizes the thickness of the inner layer which is not involved in the plastic deformation during bending (the elastic nucleus). The relation between C_0, the bending moment and beam geometry can be described

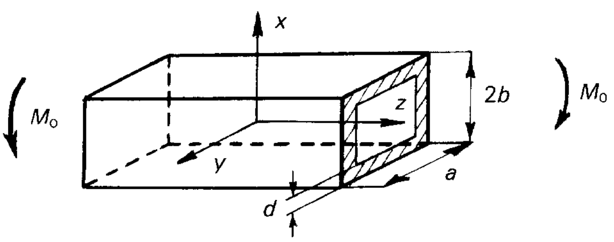

Figure 1 The structural model of a surface-modified beam; d = the thickness of the surface-modified layer; M_0 = the bending moment of the external load.

as

$$M_0 \;=\; \sigma_s ab^2[1-(1/3)(C_0/b)^2] \qquad (2)$$

From Equations 1 and 2 it follows that the residual stresses at the surface of a homogeneous beam are always below the yield stress point of the material. Plastic deformation in the inner layer of the material under the effect of residual stresses is possible only in the extreme case when $C_0 = 0$; so cyclic tests of a homogeneous beam under the effect of any constant sign moments of the external forces causing an elasto-plastic bending without the loss of the bearing capacity of the sample, should not put the material out of the "autostrengthened" state (for more details, see [5]) which would be attained during the very first loading cycle.

3. Plasticized beam (tests in a surface-active medium: $\sigma^p_{ss}/\sigma_s<1$)

The most general case was analysed when plastic deformation had taken place under bending in both the near-surface plasticized layer and part of the inner bulk zone of the beam. Solving the problem of the distribution of the stresses in a beam caused by the bending moment of the external forces included a set of equations of the elasto-plastic equilibrium, the equations of mechanical state of the material in the elastic and plastic zones, as well as boundary conditions for each of the sample zones. The solution of the set of equations only is given, rather than writing it out entirely, because of its great length

$$\sigma_{ij} \;=\; 0 \qquad \text{when } i,j \neq z \qquad (3a)$$

$$\sigma_{zz}(x) \;=\; \begin{cases} \pm\,\sigma^p_{ss} & \text{when } b-d \leqslant |x| \leqslant b \\ \pm\,\sigma_s & \text{when } C_p \leqslant |x| \leqslant b-d \\ \sigma_s \dfrac{x}{C_p} & \text{when } |x| \leqslant C_p \end{cases} \qquad (3b)$$

where C_0 is the half-width of the elastic nucleus of the beam. The choice of signs in Equation 3 is determined in the same manner as in Equation 1. The relation between the moment of the bending forces, M_p, and the value C_p is given by

$$M_p = \sigma_s ab^2\left\{\left(1-\frac{d}{b}\right)^2 - \frac{1}{3}\left(\frac{C_p}{b}\right)^2 + \frac{\sigma^p_{ss}}{\sigma_s}\left[\frac{2d}{b} - \left(\frac{d}{b}\right)^2\right]\right\} \qquad (4)$$

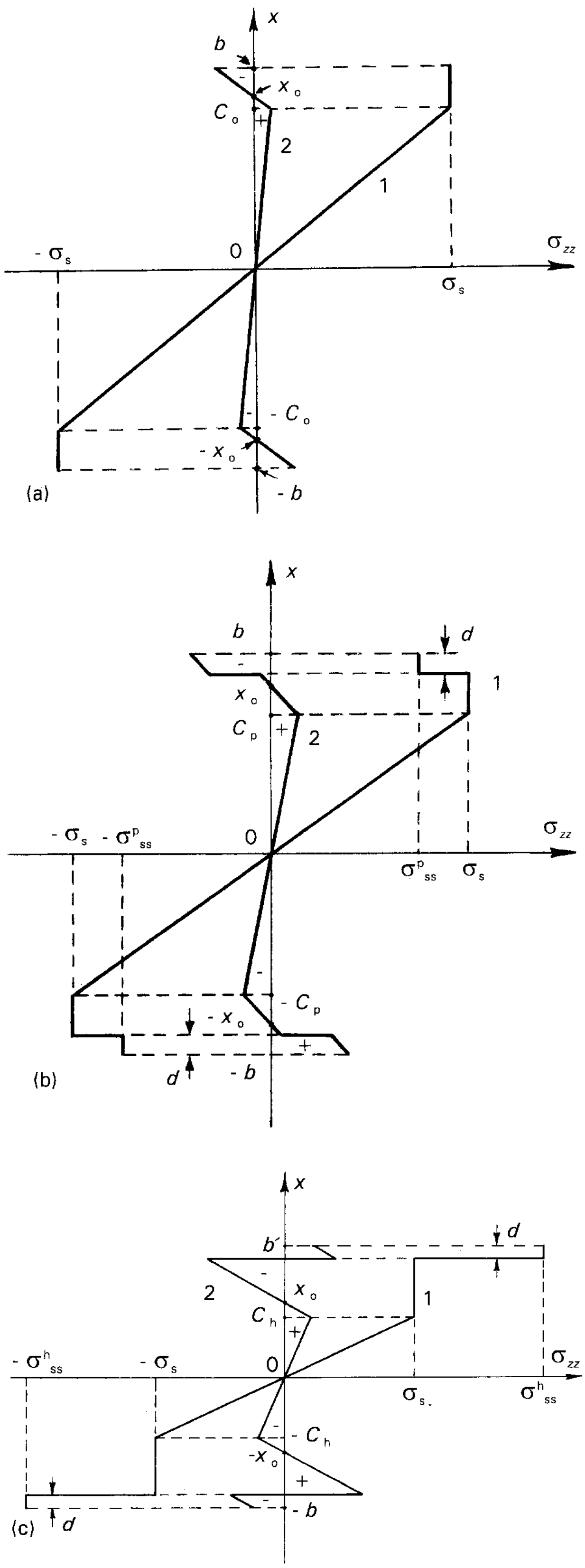

Figure 2 Distribution of (1) the external loading stresses and (2) residual stresses in a (a) homogeneous, (b) surface-plasticized, and (c) surface-hardened beam in the process of bending around the $0Y$ axis. The relation of the active bending moments M_o: M_p: $M_h = 1:1:1.4$.

Comparison of equations 2 and 4 shows that the depth of the layer involved in the plastic deformation during bending of a plasticized beam exceeds that of a homogeneous one with the same geometrical sizes of the beams and the bending moments. The stress tensor component, σ_{zz}, makes a step-like jump at the boundary between the plasticized layer and the inner bulk zone (with $|x| = b - d$) in accordance with the accepted model of the material. The value of the jump is $|\Delta\sigma_{zz}| = \sigma_s - \sigma_{ss}$.

The stress distribution in the beam can be described by

$$\sigma^r_{ij} = 0 \quad \text{when } i, j \neq z \tag{5a}$$

$$\sigma^r_{ss}(x) = \begin{cases} \pm\,\sigma^p_{ss} - \dfrac{M_p}{J}x & \text{when } b - d \leqslant |x| \leqslant b \\ \pm\,\sigma_s - \dfrac{M_p}{J}x & \text{when } C_p \leqslant |x| \leqslant b - d \\ \left(\dfrac{\sigma_s}{C_p} - \dfrac{M_p}{J}\right)x & \text{when } |x| \leqslant C_p \end{cases} \tag{5b}$$

Fig. 2b shows an example of the residual stress distribution in a plasticized beam with $a/b = 0$, 1 and $\sigma^p_{ss}/\sigma_s = 0.75$ after loading with a bending moment equal to that in a homogeneous beam (cf. Fig. 2a). Comparison of Equations 1 and 5 allows the conclusion to be drawn that the general character of the distribution of the residual stresses in the beam bulk is qualitatively similar in both cases. The extreme values of the residual stresses in the variants considered above are attained at the beam surface and at the elastic nucleus boundaries (when $x = \pm b$; $\pm C_p$). At the same time, the general level of the residual stresses, as well as their gradients in a plasticized beam, is considerably higher in comparison with a homogeneous one.

Analysis of Equations 4 and 5 also shows that under the conditions

$$b \gg d \tag{6a}$$

and

$$\sigma^p_{ss}/\sigma_s \leqslant 0{\cdot}75\,[1 - (1/3)(C_p/b)^2] \tag{6b}$$

the residual stresses in the plasticized layer of the beam attain the yield point, thus resulting in a repeated plastic deformation of the material. As this takes place, the sign of the repeated plastic deformation is opposite to the sign of the primary plastic deformation occurring in this layer under an active loading of the sample. Thus, in the case under discussion, the degree of total plastic deformation in the near-surface layer

$$e = \int_0^g (2\xi_{ij}\xi_{ij})^{1/2}\,dt$$

(where ξ_{ij} is the tensor of the deformation velocities) will grow continuously and the "autostrengthened" state cannot be obtained while loading the beam repeatedly up to the primary level. In other words, the fatigue strength of a plasticized beam under bending will be markedly lower than that of a homogeneous one loaded by the moment of the external forces of the same value as for the first beam.

4. Surface-hardened beam (tests in an active medium, with $\sigma^h_{ss}/\sigma_s > 1$)

As in the previous variant, the most general case of an elasto-plastic bending was studied when both the near-surface hardened layer and a certain part of the material in the inner bulk zone were involved in the plastic deformation caused by the bending moment, M_h. The calculations have shown that this takes place under the condition of $C_h \leqslant (b - d)\sigma_s/\sigma^h_{ss}$ where C_h is the halfwidth of the elastic nucleus. The relation between the values C_h and M_h is given by an equation of the type of Equation 4, where the parameter C_p should be replaced by C_h. The residual stress distribution in the beam here takes the form

$$\sigma^r_{ij} = 0 \quad \text{when } i, j \neq z \tag{7a}$$

$$\sigma^r_{zz}(x) = \begin{cases} \pm\,\sigma^h_{ss} - \dfrac{M_h}{J}x & \text{when } b - d \leqslant |x| \leqslant b \\ \pm\,\sigma_s - \dfrac{M_h}{J}x & \text{when } C_h \leqslant |x| \leqslant b - d \\ \left(\dfrac{\sigma_s}{C_h} - \dfrac{M_h}{J}\right)x & \text{when } |x| \leqslant C_h \end{cases} \tag{7b}$$

The choice of sign is made as in Equation (1).

From Equation 7 it follows that with bending moments M_h of the order of $\sigma_s J/(b - d) \leqslant M_h < \sigma^h_{ss} J/(b - d)$, three layers can exist differing in residual stress signs on each side of the neutral section of a hardened beam. Such a triply repeated oscillating stress distribution (under the condition $b \gg d$) requires that the relation

$$\sigma^h_{ss}/\sigma_s > 1.5[1 - (1/3)(C_h/b)^2] \tag{8}$$

be satisfied.

From Equation 8 it follows that these residual stress oscillations for the surface-hardened beams with yield point relations of $\sigma^h_{ss}/\sigma_s > 1.5$ can exist at bending moments up to the maximum value which is attained with $C_h = 0$, and corresponds to the maximum capacity of the beam. Fig. 2c shows an example of the residual stresses for a surface-hardened beam calculated with $d/b = 0.1$; $\sigma^h_{ss}/\sigma_s = 2$ and $C_h = (1/2)(b - d)$.

The calculations show that, for the case under consideration, residual stresses attain their extreme values at the elastic nucleus surface (with $x = \pm C_h$), as well as at the boundary between the hardened near-surface layer and the inner zone where, in addition, a step-like jump in stress takes place, accompanied by the change of sign. A repeated change of the signs of the residual stresses occurs in the inner bulk of the beam at the distance $x_0 = \pm \sigma_s J/M_h$ from the neutral section.

The analysis of the solution of the problem also testifies to the fact that maximum values of residual stresses in each zone with any actual bending moments do not exceed the yield points of the appropriate layers of the material. Hence, a repeated loading of a surface-hardened beam by a bending moment up to its primary level will not cause any plastic redeformation of the material; under multiple bending tests, such a beam will immediately reach the "autostrengthening" state.

A comparison of the distributions of residual stresses arising in the material during bending of surface-hardened and plasticized beams shows that, in the general case, these distributions differ qualitatively. The main difference consists in the fact that in a hardened layer, contrary to in a plasticized one, residual stresses are coincident in sign with the external loading stresses in cases when the bending moments are not very high.

5. The effect of residual stresses on the microstructure of the material

If the beam is made of a metal containing alloy elements (representing a solid solution), after bending, diffusion currents of vacancies and alloy atoms or impurities should then arise in it, being directed in such a way that they could bring about a relaxation of residual stresses (Gorsky–Konobeyevsky effect [6, 7] termed by them, the "ascending" diffusion). In the compressed areas of the material this will result in a decrease of the amount of alloy atoms which are bigger in size than those of the basic material. An opposite picture can be observed in the stretched areas. Through a relatively simple analysis of the thermodynamics process [8], it can be shown that, when the amplitude of the interlayer oscillations of the normal component of the residual stress tensor is $|\Delta\sigma_{zz}| \cong \sigma_s \sim 2 \times 10^2 - 3 \times 10^3$ MPa, the change in the relative concentration of the alloy atoms having a volume exceeding $\delta\omega \sim 3 \times 10^{-24}\,\mathrm{cm}^3$, at room temperature, would attain the magnitude $\Delta C/C \simeq \exp\{\delta\omega\Delta\sigma_{zz}/(3kT)\} - 1 \simeq 0.1$ to 1. Such a stratification would inevitably affect the yield strength of the near-surface modified layer of the beam. In fact, because $\sigma_{ss} \sim C^{1/2}$ [9], then with $\Delta C/C \sim 1$, the value of $|\Delta\sigma_{ss}|/\sigma_{ss}$ is $\simeq 50\%$ which is compatible with the change in yield point in the near-surface layer of the beam, under the effect of surface-active media [10]. Thus, variations of the plastic characteristics of the near-surface layer of the sample under the effect of residual stresses should be superimposed directly on their primary change under the influence of decreasing or increasing free surface energy of the material in an active medium. As this takes place, both these effects suppress or intensify one another, depending on the testing procedure and the type of medium.

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The stratification of the material under the residual stress effect should inevitably result in a change of the corrosion resistance in the near-surface layer of the samples, especially in cases when the resistance is determined by the passivating capacity of the alloy elements.

While comparing the variants of the general problem considered above, it should be noted that physicochemical processes connected with the "ascending" diffusion in the near-surface layer would be most intensive at a cyclic (i.e. cycles with not too high coefficients of filling), but constant in sign, loading of a beam working in a surface-hardening medium, because in this case the residual stresses are similar in sign to the external loading stresses (Fig. 2b).

It should be noted that the main conclusions drawn in analysing the residual stresses in a beam with a rectangular cross-section are not limited just to this particular case. They remain true in the general case as well, when a beam with an arbitrary cross-section is considered.

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